Name ______________________ Class ____________ Date ____________

4-6 Reteaching

Completing the Square

Completing a perfect square trinomial allows you to factor the completed trinomial as the square of a binomial.

Start with the expression $x^2 + bx$. Add $\left(\frac{b}{2}\right)^2$. Now the expression is $x^2 + bx + \left(\frac{b}{2}\right)^2$,

which can be factored into the square of a binomial: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$.

To complete the square for an expression $ax^2 + abx$, first factor out a. Then find the value that completes the square for the factored expression.

Problem

What value completes the square for $-2x^2 + 10x$?

Think	Write
Write the expression in the form $a(x^2 + bx)$.	$-2x^2 + 10x = -2(x^2 - 5x)$
Find $\frac{b}{2}$.	$\frac{b}{2} = \frac{-5}{2} = -\frac{5}{2}$
Add $\left(\frac{b}{2}\right)^2$ to the inner expression to complete the square.	$-2\left[x^2 - 5x + \left(-\frac{5}{2}\right)^2\right] = -2\left(x^2 - 5x + \frac{25}{4}\right)$
Factor the perfect square trinomial.	$-2\left(x - \frac{5}{2}\right)^2$
Find the value that completes the square.	$-2\left(\frac{25}{4}\right) = -\frac{25}{2}$

Exercises

What value completes the square for each expression?

1. $x^2 + 2x$ **2.** $x^2 - 24x$ **3.** $x^2 + 12x$

4. $x^2 - 20x$ **5.** $x^2 + 5x$ **6.** $x^2 - 9x$

7. $2x^2 - 24x$ **8.** $3x^2 + 12x$ **9.** $-x^2 + 6x$

10. $5x^2 + 80x$ **11.** $-7x^2 + 14x$ **12.** $-3x^2 - 15x$

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4-6 Reteaching (continued)

Completing the Square

You can easily graph a quadratic function if you first write it in vertex form. Complete the square to change a function in standard form into a function in vertex form.

Problem

What is $y = x^2 - 6x + 14$ in vertex form?

Think	Write
Write an expression using the terms that contain x.	$x^2 - 6x$
Find $\frac{b}{2}$.	$\frac{b}{2} = \frac{-6}{2} = -3$
Add $\left(\frac{b}{2}\right)^2$ to the expression to Complete the square.	$x^2 - 6x + (-3)^2 = x^2 - 6x + 9$
Subtract 9 from the expression so that the equation is unchanged.	$y = x^2 - 6x + 9 + 14 - 9$
Factor the perfect square trinomial.	$y = (x - 3)^2 + 14 - 9$
Add the remaining constant terms.	$y = (x - 3)^2 + 5$

Exercises

Rewrite each equation in vertex form.

13. $y = x^2 + 4x + 3$

14. $y = x^2 - 6x + 13$

15. $y = 2x^2 + 4x - 10$

16. $y = x^2 - 2x - 3$

17. $y = x^2 + 8x + 13$

18. $y = -x^2 - 6x - 4$

19. $y = -x^2 + 10x - 18$

20. $y = x^2 + 2x - 8$

21. $y = 2x^2 + 4x - 3$

22. $y = 3x^2 - 12x + 8$

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4-7 Reteaching

The Quadratic Formula

You can solve some quadratic equations by factoring or completing the square. You can solve any quadratic equation $ax^2 + bx + c = 0$ by using the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Notice the ± symbol in the formula. Whenever $b^2 - 4ac$ is not zero, the Quadratic Formula will result in two solutions.

Problem

What are the solutions for $2x^2 + 3x = 4$? Use the Quadratic Formula.

$2x^2 + 3x - 4 = 0$ — Write the equation in standard form: $ax^2 + bx + c = 0$

$a = 2;\ b = 3;\ c = -4$ — a is the coefficient of x^2, b is the coefficient of x, c is the constant term.

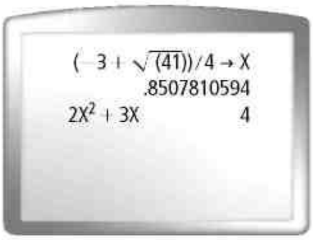
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Write the Quadratic Formula.

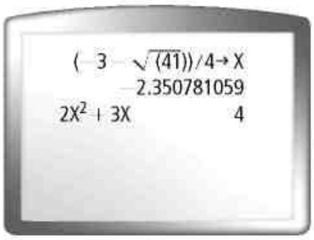
$$= \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-4)}}{2(2)}$$ Substitute 2 for a, 3 for b, and −4 for c.

$$= \frac{-3 \pm \sqrt{41}}{4}$$ Simplify.

$$= \frac{-3 + \sqrt{41}}{4} \text{ or } = \frac{-3 - \sqrt{41}}{4}$$ Write the solutions separately.

Check your results on your calculator. Replace x in the original equation with $= \frac{-3+\sqrt{41}}{4}$ and $\frac{-3-\sqrt{41}}{4}$. Both values for x give a result of 4. The solutions check.





Exercises

What are the solutions for each equation? Use the Quadratic Formula.

1. $-x^2 + 7x - 3 = 0$

2. $x^2 + 6x = 10$

3. $2x^2 = 4x + 3$

4. $4x^2 + 81 = 36x$

5. $2x^2 + 1 = 5 - 7x$

6. $6x^2 - 10x + 3 = 0$

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4-7 Reteaching (continued)

The Quadratic Formula

There are three possible outcomes when you take the square root of a real number *n*:

$n \begin{cases} > 0 \rightarrow \\ = 0 \rightarrow \\ < 0 \rightarrow \end{cases}$

two real values (one positive and one negative)

one real value (0)

no real values

Now consider the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The value under the radical symbol determines the number of real solutions that exist for the equation $ax^2 + bx + c = 0$:

$b^2 - 4ac \begin{cases} > 0 \rightarrow \\ = 0 \rightarrow \\ < 0 \rightarrow \end{cases}$

two real solutions

one real solution

no real solutions

> The value under the radical, $\boldsymbol{b^2 - 4ac}$, is called the **discriminant**.

Problem

What is the number of real solutions of $-3x^2 + 7x = 2$?

$-3x^2 + 7x = 2$	
$-3x^2 + 7x - 2 = 0$	Write in standard form.
$a = -3, b = 7, c = -2$	Find the values of *a*, *b*, and *c*.
$b^2 - 4ac$	Write the discriminant.
$(7)^2 - 4(-3)(-2)$	Substitute for *a*, *b*, and *c*.
$49 - 24$	Simplify.
25	

The discriminant, 25, is positive. The equation has two real roots.

Exercises

What is the value of the discriminant and what is the number of real solutions for each equation?

7. $x^2 + x - 42 = 0$

8. $-x^2 + 13x - 40 = 0$

9. $x^2 + 2x + 5 = 0$

10. $x^2 = 18x - 81$

11. $-x^2 + 7x + 44 = 0$

12. $\frac{1}{4}x^2 - 5x + 25 = 0$

13. $2x^2 + 7 = 5x$

14. $4x^2 + 25x = 21$

15. $x^2 + 5 = 3x$

16. $\frac{1}{9}x^2 = 4x - 36$

17. $\frac{1}{2}x^2 + 2x + 3 = 0$

18. $\frac{1}{6}x^2 = 2x + 18$

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