Chapter 5, Section 1

take note — Key Concept Standard Form of a Polynomial Function

The **standard form of a polynomial function** arranges the terms by degree in descending numerical order.

A polynomial function $P(x)$ in standard form is

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

where n is a nonnegative integer and $a_n, \ldots, a_0$ are real numbers.

$$P(x) = 4x^3 + 3x^2 + 5x - 2$$

You can classify a polynomial by its degree or by its number of terms. Polynomials of degrees zero through five have specific names, as shown in this table.

Degree	Name Using Degree	Polynomial Example	Number of Terms	Name Using Number of Terms
0		5	1	monomial
1	linear	$x + 4$	2	binomial
2		$4x^2$	1	
3	cubic	$4x^3 - 2x^2 + x$	3	trinomial
4		$2x^4 + 5x^2$	2	
5	quintic	$-x^5 + 4x^2 + 2x + 1$	4	

Got It? **1.** Write each polynomial in standard form. What is the classification of each by degree? by number of terms?

a. $3x^3 - x + 5x^4$ **b.** $3 - 4x^5 + 2x^2 + 10$

Classify each polynomial by degree and by number of terms. Simplify first if necessary.

41. $a^2 + a^3 - 4a^4$

42. 7

43. $2x(3x)$

44. $(2a - 5)(a^2 - 1)$

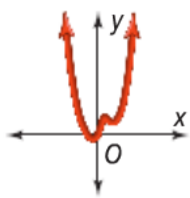
45. $(-8d^3 - 7) + (-d^3 - 6)$

46. $b(b - 3)^2$

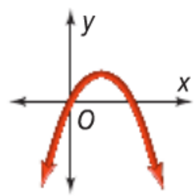
56. Reasoning A cubic polynomial function f has leading coefficient 2 and constant term 7. If $f(1) = 7$ and $f(2) = 9$, what is $f(-2)$? Explain how you found your answer.

The degree of a polynomial function affects the shape of its graph and determines the maximum number of ______________ or places where the graph changes direction. It also affects the ______________ or the directions of the graph to the far left and to the far right.

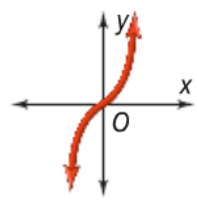
For polynomial functions of degree one or greater, there are four types of end behavior as you move to the left and move to the right, away from the origin: *up and up*, *down and down*, *down and up*, and *up and down*.



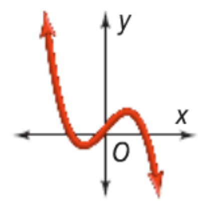
$y = x^4 - 3x^3 + 5x$



$y = -x^2 + 6x$



$y = x^3$



$y = -0.3x^3 + 4x + 2$

You can determine the end behavior of a polynomial function of degree n from the leading term ax^n of the standard form.

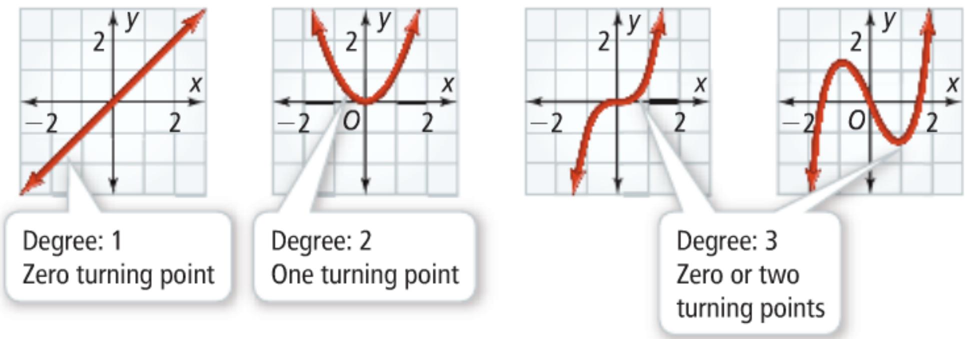
End Behavior of a Polynomial Function of Degree n With Leading Term ax^n (Moving Away From the Origin)

	Up and Up	Down and Up
	Down and Down	Up and Down

Got It? **2.** Consider the leading term of $y = -4x^3 + 2x^2 + 7$. What is the end behavior of the graph?

In general, the graph of a polynomial function of degree n ($n \geq 1$) has at most ______ turning points. The graph of a polynomial function of odd degree has an even number of turning points. The graph of a polynomial function of even degree has an odd number of turning points.

This information, combined with end behavior, determines possible shapes that the graph of a polynomial function can have.



Got It? **3.** What is the graph of each cubic function? Describe the graph.

a. $y = -x^3 + 2x^2 - x - 2$ **b.** $y = x^3 - 1$

Determine the sign of the leading coefficient and the least possible degree of the polynomial function for each graph.

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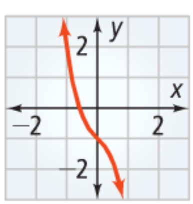

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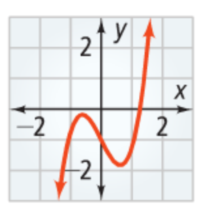

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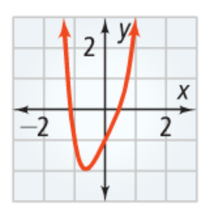


Suppose you are given a set of polynomial function outputs. You know that their inputs are an ordered set of x-values in which consecutive x-values differ by a constant. By analyzing the differences of consecutive y-values, it is possible to determine the least-degree polynomial function that could generate the data.

If the first differences are constant, the function is ______. If the second differences (but not the first) are constant, the function is ______. If the third differences (but not the second) are constant, the function is ______, and so on.

Got It? **4. a.** What is the degree of the polynomial function that generates the data shown at the right?

b. Reasoning What is an example of a polynomial function whose fifth differences are constant but whose fourth differences are not constant?

x	y
−3	23
−2	−16
−1	−15
0	−10
1	−13
2	−12
3	29

Think About a Plan The data shows the power generated by a wind turbine. The x column gives the wind speed in meters per second. The y column gives the power generated in kilowatts. What is the degree of the polynomial function that models the data?

- What are the first differences of the y-values?
- What are the second differences of the y-values?
- When are the differences constant?

x	y
5	10
6	17.28
7	27.44
8	40.96
9	58.32

54. Copy and complete the table, which shows the first and second differences in y-values for consecutive x-values for a polynomial function of degree 2.

x	y	1st diff.	2nd diff.
−3	14	−8	2
−2	6	■	2
−1	■	−4	2
0	−4	−2	2
1	■	0	2
2	−6	■	
3	■		

Chapter 5, Section 2

take note

Key Concepts **Roots, Zeros, and x-intercepts**

The following are equivalent statements about a real number b and a polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$.

- $x - b$ is a linear factor of the polynomial $P(x)$.
- b is a zero of the polynomial function $y = P(x)$.
- b is a root (or solution) of the polynomial equation $P(x) = 0$.
- b is an x-intercept of the graph of $y = P(x)$.

Got It? **2.** What are the zeros of $y = x(x - 3)(x + 5)$? Graph the function.

take note

Theorem **Factor Theorem**

The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

Got It? **3.** **a.** What is a quadratic polynomial function with zeros 3 and -3?
b. What is a cubic polynomial function with zeros 3, 3, and -3?
c. **Reasoning** Graph both functions. How do the graphs differ? How are they similar?

Write a polynomial function in standard form with the given zeros.

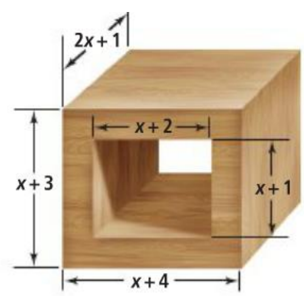
19. $x = 5, 6, 7$ **20.** $x = -2, 0, 1$ **21.** $x = -5, -5, 1$ **22.** $x = 3, 3, 3$

23. $x = 1, -1, -2$ **24.** $x = 0, 4, -\frac{1}{2}$ **25.** $x = 0, 0, 2, 3$ **26.** $x = -1, -2, -3, -4$

44. Carpentry A carpenter hollowed out the interior of a block of wood as shown at the right.

a. Express the volume of the original block and the volume of the wood removed as polynomials in factored form.

b. What polynomial represents the volume of the wood remaining?



take note

Key Concept How Multiple Zeros Affect a Graph

If a is a zero of multiplicity n in the polynomial function $y = P(x)$, then the behavior of the graph at the x-intercept a will be close to linear if ______, close to ______ if $n = 2$, close to cubic if ______, and so on.

Got It? 4. What are the zeros of $f(x) = x^3 - 4x^2 + 4x$? What are their multiplicities? How does the graph behave at these zeros?

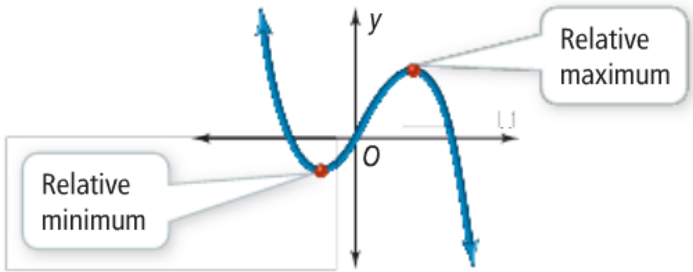
For each function, determine the zeros. State the multiplicity of any multiple zeros.

52. $f(x) = x^3 - 36x$

53. $y = (x + 1)(x - 4)(3 - 2x)$

54. $y = (x + 7)(5x + 2)(x - 6)^2$

If the graph of a polynomial function has several turning points, the function can have a [ ] and a [ ] A relative maximum is the value of the function at an up-to-down turning point. A relative minimum is the value of the function at a down-to-up turning point.



Got It? 5. What are the relative maximum and minimum of $f(x) = 3x^3 + x^2 - 5x$?

Got It? 6. What is the maximum volume of the camera in Problem 6, if the sum of the dimensions is at most 4 inches?

Find the relative maximum, relative minimum, and zeros of each function.

47. $y = 2x^3 - 23x^2 + 78x - 72$ **48.** $y = 8x^3 - 10x^2 - x - 3$ **49.** $y = (x + 1)^4 - 1$