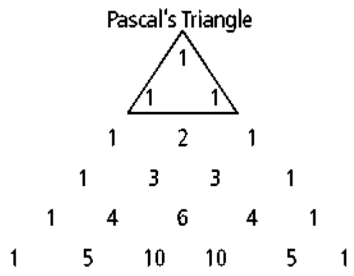
Name________________________________ Class ____________ Date __________

5-7 Reteaching

The Binomial Theorem

You can find the coefficients of a binomial expansion in Pascal's Triangle.

To create Pascal's Triangle, start by writing a triangle of 1's. This triangle forms the first two rows. Each row has one more element than the one above it. Each row begins with a 1, and then each element is the sum of the two closest elements in the row above. The last element in each row is a 1.

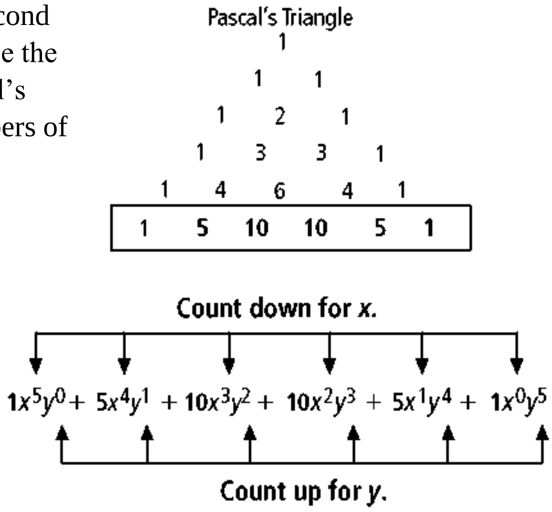


Problem

What is the expansion of $(x+y)^5$? Use Pascal's Triangle.

Step 1 The power of the binomial corresponds to the second number in each row of Pascal's Triangle. Because the power of this binomial is 5, use the row of Pascal's Triangle with 5 as the second number. The numbers of this row are the coefficients of the expansion.

Step 2 The exponents of the *x*-terms of the expansion begin with the power of the binomial and decrease to 0. The exponents of the *y*-terms of the expansion begin with 0 and increase to the power of the binomial.



Step 3 Simplify all terms to write the expansion in standard form.

$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

Exercises

Write the expansion of each binomial.

1. $(a+b)^3$

2. $(x-y)^4$

3. $(r+1)^5$

4. $(a-b)^6$

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5-7 Reteaching (continued)

The Binomial Theorem

- The *Binomial Theorem* states that for any binomial $(a + b)$ and any positive integer n,
  $(a+b)^n = {}_nC_0a^n + {}_nC_1a^{n-1}b + {}_nC_2a^{n-2}b^2 + \ldots + {}_nC_{n-1}ab^{n-1} + {}_nC_nb^n$.
- The theorem provides an effective method for expanding any power of a binomial.

Evaluate the combination ${}_nC_k$ as $\dfrac{n!}{k!(n-k)!}$.

Problem

What is the expansion of $(3x + 2)^3$? Use the Binomial Theorem.

Step 1 Determine a, b, and n.

$a = 3x$, $b = 2$, $n = 3$

Step 2 Use the formula to write the equation.

$$(3x+2)^3 = {}_3C_0(3x)^3 + {}_3C_1(3x)^2(2) + {}_3C_2(3x)(2)^2 + {}_3C_3(2)^3$$

Step 3 Simplify.

$$= 1(27x^3) + 3(9x^2)(2) + 3(3x)(4) - 1(8)$$
$$= 27x^3 + 54x^2 + 36x + 8$$

Exercises

Fill in the correct coefficients, variables, and exponents for the expanded form of each binomial.

5. $(x + y)^4 = x\square + \square x^3y + 6x\square y^2 + \square xy\square + \square^4$

6. $(z - y)^3 = z\square - \square z^2y + \square zy\square - \square^3$

7. $(x + z)^5 = x\square + \square x^4z + 10x\square z^2 + \square x^2z\square + \square xz^4 + \square^5$

Write the expansion of each binomial. Use the Binomial Theorem.

8. $(x + y)^5$

9. $(x - y)^5$

10. $(2x + y)^3$

11. $(x + 3y)^4$

12. $(x - 2y)^5$

13. $(2x - y)^5$

14. $(x - 3y)^4$

15. $(4x - y)^3$

16. $(x - 1)^5$

17. $(1 - x)^3$

18. $(x^2 + 1)^3$

19. $(y^2 + a)^4$

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5-8 Reteaching

Polynomial Models in the Real World

Problem

What polynomial function has a graph that passes through the four points (0, −1), (−1, −7), (2, 17), and (−2, −27)?

Step 1 Use the $(n + 1)$ Point Principle to determine the degree (n) of the polynomial that fits these points perfectly. In this case $n + 1 = 4$, so the degree of the polynomial is 3 and the polynomial that fits these points will be $y = ax^3 + bx^2 + cx + d$.

Step 2 Substitute the x- and y-values from the four points given in the problem. Now you have four linear equations in four unknowns (a, b, c, d).

$a(0)^3 + b(0)^2 + c(0) + d = -1$ or $0a + 0b + 0c + d = -1$

$a(-1)^3 + b(-1)^2 + c(-1) + d = -7$ or $-a + b - c + d = -7$

$a(2)^3 + b(2)^2 + c(2) + d = 17$ or $8a + 4b + 2c + d = 17$

$a(-2)^3 + b(-2)^2 + c(-2) + d = -27$ or $-8a + 4b - 2c + d = -27$

Step 3 Enter the coefficients from the four linear equations as a matrix in your graphing calculator.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & -1 & 1 & -7 \\ 8 & 4 & 2 & 1 & 17 \\ -8 & 4 & -2 & 1 & -27 \end{bmatrix}$$

Step 4 Recall that the **rref** function on your calculator returns the reduced row echelon form of a matrix. The last column of the reduced row echelon form is the solution to the system of equations represented by the matrix.

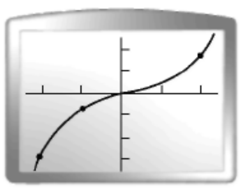
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

Use the **rref** function to determine the coefficient values a, b, c, d.

$a = 2,\ b = -1,\ c = 3,\ d = -1$

So, the polynomial function is $y = 2x^3 - x^2 + 3x - 1$.

Step 5 Use the CubicReg calculator function with the four original points to check your answer.



Exercises

Find the polynomial function that passes through each set of points.

1. (1, –4), (−2, 38), (0, 2), and (−1, 10)

2. (1, –5), (−3, 39), and (0, –6)

5-8 Reteaching (continued)

Polynomial Models in the Real World

Problem

The table shows winning times in the 400-meter run at a state track meet.

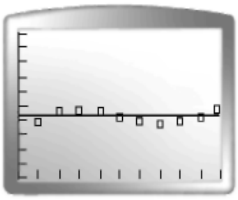
Track and field 400-Meter Run										
Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Seconds	53.86	54.66	54.72	54.60	54.27	53.87	53.70	53.89	54.14	54.62

Let x = the number of years since 2001 and y = the number of seconds.

Determine whether a linear model, a quadratic model, or a cubic model best fits the data. Then use the model to estimate the winning time in 2018. Does your answer seem reasonable?

Find and graph a linear model, a quadratic model, and a cubic model for the data.

Linear model



$y = -0.02455x + 54.36800$

Quadratic model

$y = 0.0913x^2 - 0.12496x + 54.56883$

Cubic model

$y = 0.01961x^3 - 0.31443x^2 + 1.36733x + 52.88633$

Since the coefficient of the x^2-term in the quadratic model is almost 0, the graph of the quadratic model is very similar to the graph of the linear model.

The cubic model appears to be the best fit. Use it to estimate the winning time in 2013, or when $x = 12$. Substitute 12 for x and simplify.

$y = 0.01961(12)^3 - 0.31443(12)^2 + 1.36733(12) + 52.88633 \approx 57.9$

This estimate does seem to be reasonable because the times are in the mid-50's.

Exercises

3. The table shows winning points in men's springboard diving.

Men's Olympic Springboard Diving Records							
Year	1980	1984	1988	1992	1996	2000	2004
Points	905.02	754.41	730.80	676.53	701.46	708.72	787.38

SOURCE: *www.infoplease.com*

a. Find a linear, quadratic, and cubic model for the data. Which model best fits the data?

b. Use the model of best fit to estimate the diving record in 2018. Does your estimate seem reasonable?