

5-3

Reteaching

Solving Polynomial Equations

Problem

What are the real or imaginary solutions of the polynomial equation

$$2x^3 + 16 = 0?$$

$$2x^3 + 16 = 0$$

$$2(x^3 + 8) = 0$$

$$2(x + 2)(x^2 - 2x + 4) = 0$$

$$x + 2 = 0 \text{ or } x^2 - 2x + 4 = 0$$

$$x = -2 \text{ or } x = \frac{2 \pm \sqrt{4 - 4(1)(4)}}{2(1)}$$

$$x = -2 \text{ or } x = \frac{2 \pm 2i\sqrt{3}}{2}$$

$$x = -2 \text{ or } x = 1 \pm i\sqrt{3}$$

Factor out the GCF. In this case, it is 2.

Factor the remaining cubic expression.

Use the Zero-Product Property.

Solve each equation for x . Use the Quadratic Formula when necessary.

Simplify.

The solutions are -2 and $1 \pm i\sqrt{3}$.

Exercises

Find the real or imaginary solutions of each polynomial equation.

1. $x^3 - 8 = 0$

2. $4x^3 + 4 = 0$

3. $x^4 - x^2 - 72 = 0$

4. $x^4 + 9x^2 = -20$

5. $x^4 - 27x = 0$

6. $8x^3 = -1$

7. $7x^4 = -28x^2 - 21$

8. $x^3 = 64$

9. $8x^3 + 27 = 0$

10. $x^4 - 7x^2 = -12$

11. $2x^4 + 16x^2 = 40$

12. $2x^4 - 16x = 0$

13. $9x^4 - 25 = 0$

14. $2x^4 - x^2 = 3$

15. $x^4 + 5x^2 = -4$

16. $x^4 - 7x^2 - 8 = 0$

17. $2x^3 + 16 = 0$

18. $x^4 - 5x^2 - 24 = 0$

5-3**Reteaching** (continued)**Solving Polynomial Equations****Problem**

You have a brother and a sister. Your brother is 3 years older than you. Your sister is 2 years older than your brother. The product of all three ages is 3744. How old are you and your siblings?

Step 1 Define the variables.

Let your age = x .

Your brother's age = $x + 3$.

Your sister's age = $x + 3 + 2 = x + 5$.

Step 2 Write an equation.

Because the product of all three ages equals 3744, $x(x + 3)(x + 5) = 3744$.

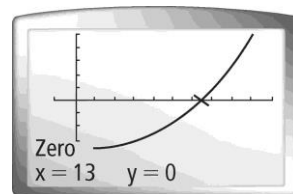
Step 3 Rewrite the equation in $P(x) = 0$ form.

$$(x^2 + 3x)(x + 5) = 3744$$

$$x^3 + 3x^2 + 5x^2 + 15x = 3744$$

$$x^3 + 8x^2 + 15x - 3744 = 0$$

Step 4 Using a graphing calculator, graph the equation. Use the Zero feature to solve for x .



Step 5 Once you have the value of x (your age), you can solve for the other ages. Since $x = 13$, your brother's age is $x + 3 = 16$. Your sister's age is $x + 5 = 18$.

Exercises

19. A slice of wood 3 in. thick is cut of a cube of wood. The remaining solid has a volume of 320 in.^2 . What are the dimensions of the original block of wood?
20. The water level in a rectangular fish tank is 4 in. from the top. The depth of the water is the same as the width of the tank, which is half of its length. The volume of the water in the tank is 4394 in.^3 . What is the volume of the fish tank?

5-4 **Reteaching**

Dividing Polynomials

Problem

What is the quotient and remainder? Use polynomial long division to divide $2x^2 + 6x - 7$ by $x + 1$.

Step 1 To find the first term of the quotient, divide the highest-degree term of $2x^2 + 6x + 7$ by the highest-degree term of the divisor, $x + 1$. Circle these terms before dividing.

$$\begin{array}{r} 2x \\ (x) + 1 \overline{) 2x^2 + 6x + 7} \end{array}$$

Step 2 Multiply $x + 1$ by the new term, $2x$, in the quotient. $2x(x + 1) = 2x^2 + 2x$. Align like terms.

$$\begin{array}{r} 2x \\ x + 1 \overline{) 2x^2 + 6x + 7} \\ \underline{2x^2 + 2x} \end{array}$$

Step 3 Subtract to get $4x$. Bring down the next term, 7 .

$$\begin{array}{r} 2x \\ x + 1 \overline{) 2x^2 + 6x + 7} \\ \underline{2x^2 + 2x} \\ 4x + 7 \end{array}$$

Step 4 Divide the highest-degree term of $4x + 7$ by the highest-degree term of $x + 1$. Circle these terms before dividing.

$$\begin{array}{r} 2x + 4 \\ (x) + 1 \overline{) 2x^2 + 6x + 7} \\ \underline{2x^2 + 2x} \\ 4x + 7 \\ \underline{4x + 4} \\ 3 \end{array}$$

Step 5 Repeat Steps 2 and 3. The *remainder* is 3 because its degree is less than the degree of $x + 1$.

$2x^2 + 6x + 7$ divided by $x + 1$ is $2x + 4$, with a remainder of 3. The quotient is $2x + 4$ with remainder 3.

Check the answer by multiplying $(x + 1)$ by $(2x + 4)$ and adding 3.
 $(x + 1)(2x + 4) + 3 = 2x^2 + 6x + 7$

Exercises

Divide using polynomial long division.

1. $(3x^2 - 8x + 7) \div (x - 1)$

2. $(x^3 + 5x^2 - 3x - 4) \div (x + 6)$

3. $(x^2 + 3x - 8) \div (x - 5)$

4. $(x^2 + 6x + 14) \div (x + 3)$

5. $(x^3 - 7x^2 + 11x + 3) \div (x - 3)$

6. $(2x^3 - 3x^2 - x - 2) \div (x - 2)$

7. $(2x^2 - 4x + 7) \div (x - 3)$

8. $(x^3 + 2x^2 - 20x + 4) \div (x + 7)$

9. $(x^2 - 5x + 2) \div (x - 1)$

10. $(2x^3 + 3x^2 + x + 6) \div (x + 3)$

5-4 **Reteaching** (continued)

Dividing Polynomials

Problem

Use synthetic division to divide $x^3 + 13x^2 + 46x + 48$ by $x + 3$. What is the quotient and remainder?

Step 1 Set up your polynomial division.

$$(x^3 + 13x^2 + 46x + 48) \div (x + 3)$$

Step 2 Reverse the sign of the constant, 3, in the divisor.
Write the coefficients of the dividend: 1 13 46 48.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ \hline \end{array}$$

Step 3 Bring the first coefficient, 1, down to the bottom line.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ \hline & 1 & & & \end{array}$$

Step 4 Multiply the coefficient, 1, by the divisor, -3 . Put this product, -3 , underneath the second coefficient 13, and add those two numbers: $13 + (-3) = 10$.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ & & -3 & & \\ \hline & 1 & 10 & & \end{array}$$

Step 5 Continue multiplying and adding through the last coefficient. The final sum is the remainder.

$$\begin{array}{r|rrrr} -3 & 1 & 13 & 46 & 48 \\ & & -3 & -30 & -48 \\ \hline & 1 & 10 & 16 & 0 \end{array}$$

The quotient is $x^2 + 10x + 16$. Since the remainder is 0, $x + 3$ is a factor of $x^3 + 13x^2 + 46x + 48$.

Exercises

What is the quotient and remainder of the following polynomials?

11. $(x^3 - 2x + 8) \div (x + 2)$

12. $(12x^3 - 71x^2 + 57x - 10) \div (x - 5)$

13. $(3x^4 + x^3 - 6x^2 - 9x + 12) \div (x + 1)$

14. $(2x^3 - 15x + 23) \div (x - 2)$

15. $(x^3 + x + 10) \div (x + 2)$

16. $(x^4 - 12x^3 - 18x^2 + 10) \div (x + 4)$