Name ______________________ Class ____________ Date ____________

5-1 Reteaching

Polynomial Functions

Problem

What is the classification of the following polynomial by its degree? by its number of terms? What is its end behavior? $5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

Step 1 Write the polynomial in standard form. First, combine any like terms. Then, place the terms of the polynomial in descending order from greatest exponent value to least exponent value.

$5x^4 - 3x + 4x^6 + 9x^3 - 12 - x^6 + 3x^4$

$8x^4 - 3x + 3x^6 + 9x^3 - 12$ Combine like terms.

$3x^6 + 8x^4 + 9x^3 - 3x - 12$ Place terms in descending order.

Step 2 The degree of the polynomial is equal to the value of the greatest exponent. This will be the exponent of the first term when the polynomial is written in standard form.

$3x^6 + 8x^4 + 9x^3 - 3x - 12$ The first term is $3x^6$.

$3x^6$ The exponent of the first term is 6.

This is a sixth-degree polynomial.

Step 3 Count the number of terms in the simplified polynomial. It has 5 terms.

Step 4 To determine the end behavior of the polynomial (the directions of the graph to the far left and to the far right), look at the degree of the polynomial (n) and the coefficient of the leading term (a).

If a is positive and n is even, the end behavior is up and up.

If a is positive and n is odd, the end behavior is down and up.

If a is negative and n is even, the end behavior is down and down.

If a is negative and n is odd, the end behavior is up and down.

The leading term in this polynomial is $3x^6$.
a (+3) is positive and n (6) is even, so the end behavior is up and up.

Exercises

What is the classification of each polynomial by its degree? by its number of terms? What is its end behavior?

1. $8 - 6x^3 + 3x + x^3 - 2$ **2.** $15x^7 - 7$ **3.** $2x - 6x^2 - 9$

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5-1 Reteaching (continued)

Polynomial Functions

Problem

What is the degree of the polynomial function that generates the data shown at the right? What are the differences when they are constant? To find the degree of a polynomial function from a data table, you can use the differences of the *y*-values.

x	*y*
–3	52(y_1)
–2	18(y_2)
–1	2(y_3)
0	–2 (y_4)
1	0 (y_5)
2	2 (y_6)
3	–2 (y_7)

Step 1 Determine the values of $y_2 - y_1$, $y_3 - y_2$, $y_4 - y_3$, $y_5 - y_4$, $y_6 - y_5$, $y_7 - y_6$. These are called the first differences. Make a new column using these values.

Step 2 Continue determining differences until the *y*-values are all equal. The quantity of differences is the degree of the polynomial function.

The third differences are all equal so this is a third degree polynomial function. The value of the third differences is –6.

x	*y*	1st diff.
−3	52 (y_1)	
		−34 (y_8)
−2	18 (y_2)	
		−16 (y_9)
−1	2 (y_3)	
		−4 (y_{10})
0	−2 (y_4)	
		2 (y_{11})
1	0 (y_5)	
		2 (y_{12})
2	2 (y_6)	
		−4 (y_{13})
3	−2 (y_7)	

x	*y*	1st diff.	2nd diff.	3rd diff.
−3	52 (y_1)	−34 (y_8)	18	−6
−2	18 (y_2)	−16 (y_9)	12	−6
−1	2 (y_3)	−4 (y_{10})	6	−6
0	−2 (y_4)	2 (y_{11})	0	−6
1	0 (y_5)	2 (y_{12})	−6	
2	2 (y_6)	−4 (y_{13})		
3	−2 (y_7)			

Exercises

What is the degree of the polynomial function that generates the data in the table? What are the differences when they are constant?

4.

x	*y*
–3	216
–2	24
–1	0
0	0
1	0
2	–24
3	–216

5.

x	*y*
–3	–101
–2	–37
–1	–11
0	–5
1	–1
2	19
3	73

6.

x	*y*
–3	6
–2	26
–1	8
0	0
1	2
2	–34
3	–204

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5-2 Reteaching

Polynomials, Linear Factors, and Zeros

The Factor Theorem tells you that if you know the zeros of a polynomial function, you can write the polynomial.

Factor Theorem

The expression $x - a$ is a factor of a polynomial if and only if the value a is a zero of the related polynomial function.

Problem

What is a cubic polynomial function in standard form with zeros 0, 4, and −2?

Each zero (a) is part of a linear factor of the polynomial, so you can write each factor as $(x - a)$.

$(x - a_1)(x - a_2)(x - a_3)$	Set up the cubic polynomial factors.
$a_1 = 0, a_2 = 4, a_3 = -2$	Assign the zeros.
$(x - 0)(x - 4)[x - (-2)]$	Substitute the zeros into the factors.
$f(x) = x(x - 4)(x + 2)$	Write the polynomial function in factored form.
$f(x) = x(x^2 - 2x - 8)$	Multiply $(x - 4)(x + 2)$.
$f(x) = x^3 - 2x^2 - 8x$	Multiply by x using the Distributive Property.

The polynomial function written in standard form is $f(x) = x^3 - 2x^2 - 8x$.

Exercises

Write a polynomial function in standard form with the given zeros.

1. 5,−1, 3

2. 1, 7,−5

3. −1, 1,−6

4. 2,−2,−3

5. 2, 1, 3

6. 2, 3,−3,−1

7. 0,−8, 2

8. −10, 0, 2

9. $-2, 2, -\frac{3}{2}$

10. $-1, \frac{2}{3}$

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5-2 Reteaching (continued)

Polynomials, Linear Factors, and Zeros

You can use a polynomial function to find the minimum or maximum value of a function that satisfies a given set of conditions.

Problem

Your school wants to put in a swimming pool. The school wants to maximize the volume while keeping the sum of the dimensions at 40 ft. If the length must be 2 times the width, what should each dimension be?

Step 1 First, define a variable x. Let x = the width of the pool.

Step 2 Determine the length and depth of the pool using the information in the problem.

The length must be 2 times the width, so length = $2x$.

The length plus width plus depth must equal 40 ft, so depth = $40 - x - 2x = 40 - 3x$.

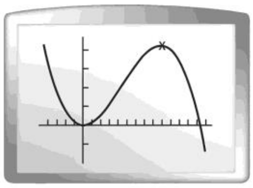
Step 3 Create a polynomial in standard form using the volume formula

V = length · width · depth

$= 2x(x)(40 - 3x)$

$= -6x^3 + 80x^2$

Step 4 Graph the polynomial function. Use the MAXIMUM feature. The maximum volume is 2107 ft^3 at a width of 8.9 ft.



Maximum
X = 8.8888882 Y = 2106.9959

Step 5 Evaluate the remaining dimensions: width = $x \approx 8.9$ ft

length = $2x \approx 17.8$ ft

depth = $40 - 3x \approx 13.3$ ft

Exercises

11. Find the dimensions of the swimming pool if the sum must be 50 ft and the length must be 3 times the depth.

12. Find the dimensions of the swimming pool if the sum must be 40 ft and the depth must be one tenth of the length.

13. Find the dimensions of the swimming pool if the sum must be 60 ft and the length and width are equal.