Composition of Functions

I. Model Problems

In this example we will evaluate a function for a given value of *x*.

Example 1: If $f(x) = 5x + 3$ and $g(x) = 3x^2$, find $f(g(4))$.

Substitute 4 for *x* for the function $g(x)$. Simplify.

$$f(g(4))$$
$$f(3 \cdot 4^2)$$
$$f(3 \cdot 16)$$

Substitute 48 for *x* for the function $f(x)$. Simplify.

$$f(48)$$
$$5(48) + 3$$
$$240 + 3$$
$$243$$

Answer: $f(g(4)) = 243$

<u>A word on notation</u>

$f \circ g(4)$ is another way of expressing $f(g(4))$

Practice Problems start on the next page

II. Practice Problems

Evaluate each composition below

1. If $f(x) = 2x + 7$ and $g(x) = 3x - 2$, find $f(g(6))$.
2. If $f(x) = 2x + 7$ and $g(x) = 3x - 2$, find $g \circ f(6)$
3. If $f(x) = -5x + 2$ and $g(x) = \frac{1}{2}x + 4$, find $f(g(12))$.
4. If $f(x) = -5x + 2$ and $g(x) = \frac{1}{2}x + 4$, find $g(f(12))$.
5. If $g(x) = -3x^2 + 6$ and $h(x) = 9x + 3$, find $g \circ h\left(\frac{1}{3}\right)$.
6. If $g(x) = -3x^2 + 6$ and $h(x) = 9x + 3$, find $h\left(g\left(\frac{1}{3}\right)\right)$.
7. If $f(x) = -\frac{1}{5}x + 4$ and $g(x) = 4x^2$, find $f \circ g(10)$.
8. If $f(x) = -\frac{1}{5}x + 4$ and $g(x) = 4x^2$, find $g \circ f(10)$.
9. If $f(x) = 4x - 7$ and $g(x) = |2x - 9|$, find $f(g(-6))$.
10. If $f(x) = 4x - 7$ and $g(x) = |2x - 9|$, find $g(f(-6))$.
11. If $g(x) = 3|x - 4| + 6$ and $h(x) = -x^3$, find $g \circ h(4)$.
12. If $g(x) = 3|x - 4| + 6$ and $h(x) = -x^3$, find $h(g(4))$.
13. If $f(x) = \sqrt{x + 2}$ and $g(x) = |2x|$, find $f \circ g(-7)$.
14. If $f(x) = \sqrt{x + 2}$ and $g(x) = |2x|$, find $g(f(-7))$.
15. If $g(x) = 2x^3 - 4$ and $h(x) = \sqrt{x + 6}$, find $g(h(19))$.
16. If $g(x) = 2x^3 - 4$ and $h(x) = \sqrt{x + 6}$, find $h \circ g(4)$.
17. If $f(x) = |7x + 4|$ and $g(x) = \sqrt{3x} = 6$, find $f(g(12))$.
18. If $g(x) = 3x^3 - 2x^2 + 4$ and $h(x) = |6x + 15|$, find $g \circ h(-3)$.
19. If $f(x) = -3x + 2$, $g(x) = 2x^2$, and $h(x) = 4|7 - x| + 6$, find $f\left(g(h(1))\right)$.
20. If $f(x) = -3x$, $g(x) = \sqrt{2x}$, and $h(x) = |4x| - 12$, find $f\left(h(g(18))\right)$.

21. Are compositions commutative?

Commutative means that order does not matter. Multiplication is commutative because 2*3 = 3*2. In either case, we get 6

Does f(g(x)) = g(f(x))?

If you're stuck, try some specific examples with the functions below.
Does f(g(1)) = g(f(1))?
$f(x) = x^2 + 1$
g(x) = 3x

Challenge Problems

1. If $f(x) = x^2$ and $g(x) = -12x + 7$, find the domain and range of $f(x)$, $g(x)$,and $f(g(x))$.
2. If $g(x) = -2x^2 - 5x$ and $h(x) = 3x + 2$, find $g(h(x))$.
3. Find the error in the student's work for the following problem:

   If $f(x) = x^2 - 3$ and $g(x) = 5x$, find $f(g(-3))$.

   $g(x) = 5x$ $\qquad$ $f(-3) = (-3)^2 - 3$

   $g(-3) = 5(-3)$ $\qquad$ $f(-3) = 9 - 3$

   $g(-3) = -15$ $\qquad$ $f(-3) = 6$

   $(-15)(6)$

   -90

4. Two functions are inverses of each other if $f(g(x)) = x$ and $g(f(x)) = x$. If $f(x) = x + 3$, find its inverse $g(x)$
5. A toy manufacturer has a new product to sell. The number of units to be sold, *x*, is a function of the price *p*: $n(p) = 30 - 25p$. The revenue earned is a function of the number of units sold: $r(x) = 1000 - \frac{1}{4}x^2$. Find the function for revenue in terms of price, *p*.

 Name_____

Function Operations

Date_____ Period____

Perform the indicated operation.

1) $g(n) = n^2 + 4 + 2n$
$h(n) = -3n + 2$
Find $(g \cdot h)(1)$

2) $f(x) = 4x - 3$
$g(x) = x^3 + 2x$
Find $(f - g)(4)$

3) $h(x) = 3x + 3$
$g(x) = -4x + 1$
Find $(h + g)(10)$

4) $g(a) = 3a + 2$
$f(a) = 2a - 4$
Find $\left(\frac{g}{f}\right)(3)$

5) $g(x) = 2x - 5$
$h(x) = 4x + 5$
Find $g(3) - h(3)$

6) $g(a) = 2a - 1$
$h(a) = 3a - 3$
Find $(g \cdot h)(-4)$

7) $g(t) = t^2 + 3$
$h(t) = 4t - 3$
Find $(g \cdot h)(-1)$

8) $g(n) = 3n + 2$
$f(n) = 2n^2 + 5$
Find $g(f(2))$

9) $g(x) = -x^2 - 1 - 2x$
$f(x) = x + 5$
Find $(g - f)(x)$

10) $f(x) = 3x - 1$
$g(x) = x^2 - x$
Find $\left(\frac{f}{g}\right)(x)$

11) $g(a) = -3a - 3$
$f(a) = a^2 + 5$
Find $(g - f)(a)$

12) $h(t) = 2t + 1$
$g(t) = 2t + 2$
Find $(h - g)(t)$

13) $f(x) = 2x^3 - 5x^2$
$g(x) = 2x - 1$
Find $(f \cdot g)(x)$

14) $h(n) = 4n + 5$
$g(n) = 3n + 4$
Find $(h - g)(n)$

15) $g(a) = -3a^2 - a$
$h(a) = -2a - 4$
Find $\left(\frac{g}{h}\right)(a)$

16) $f(n) = 2n$
$g(n) = -n - 4$
Find $(f \circ g)(n)$

17) $h(a) = 3a$
$g(a) = -a^3 - 3$
Find $\left(\frac{h}{g}\right)(a)$

18) $g(n) = 2n + 3$
$h(n) = n - 1$
Find $(g \circ h)(n)$

19) $h(x) = x^2 - 2$
$g(x) = 4x + 1$
Find $(h \circ g)(x)$

20) $g(t) = 2t + 5$
$f(t) = -t^2 + 5$
Find $(g + f)(t)$

21) $g(x) = 2x - 2$
$f(x) = x^2 + 3x$
Find $(g \circ f)(-2 + x)$

22) $g(a) = 2a + 2$
$h(a) = -2a - 5$
Find $(g \circ h)(-4 + a)$

Inverse Functions

The function $g(x)$ are inverses of each other $f(x)$ if $g(f(x)) = x$ and $g(f(x)) = x$.

The inverse of the function $f(x)$ is indicated with the notation $f^{-1}(x)$, read *f* inverse (this notation does **not** mean $\frac{1}{f(x)}$).

I. Model Problems

In this example we will find the inverse of a discrete function for a given as a list of ordered pairs.

Example 1: If $\boldsymbol{f = \{(3,2),\ (4,-6),(-2,11),(5,5)\}}$ ***find*** $\boldsymbol{f^{-1}(x)}$**.**

When finding the inverse exchange *x* and *y*. The ordered pairs (*x*,*y*) become (*y*,*x*).

$f = \{(3,2),\ (4,-6),(-2,11),(5,5)\}$

$f^{-1} = \{(2,3),\ (-6,4),(11,-2),(5,5)\}$

Answer: $\boldsymbol{f^{-1} = \{(2,3),\ (-6,4),(11,-2),(5,5)\}}$

In these examples we will find the inverse of functions given as an equation.

Example 2: If $\boldsymbol{f(x) = 3x + 10}$ ***find*** $\boldsymbol{f^{-1}(x)}$**.**

Write function in terms of *y*.

$$f(x) = 3x + 10$$

$$y = 3x + 10$$

When finding the inverse exchange *x* and *y*.

$$x = 3y + 10$$

Solve for *y*.

$$-10 \qquad -10$$

$$\frac{x - 10}{3} = \frac{3y}{x}$$

$$\frac{x - 10}{3} = y$$

Rewrite as $f^{-1}(x)$.

$$f^{-1}(x) = \frac{x - 10}{3}$$

Answer: $\boldsymbol{f^{-1}(x) = \frac{x-10}{3}}$

Example 3: If $\boldsymbol{f(x) = \sqrt{x + 12}}$ ***find*** $\boldsymbol{f^{-1}(x)}$**.**

Write function in terms of *y*.

$$f(x) = \sqrt{x + 12}$$

$$y = \sqrt{x + 12}$$

When finding the inverse exchange *x* and *y*.

$$x = \sqrt{y + 12}$$

Solve for *y*. Square both sides of the equation.

$$x^2 = \left(\sqrt{y + 12}\right)^2$$

$$x^2 = y + 12$$

$$-12 \qquad -12$$

$$x^2 - 12 = y$$

Rewrite as $f^{-1}(x)$.

$$f^{-1}(x) = x^2 - 12$$

Answer: $\boldsymbol{f^{-1}(x) = x^2 - 12,}$ $x \in R \mid x \geq 0$

II. Practice Problems

Solve.

1. Is $g(x) = \frac{1}{2}x - 2$ the inverse of $f(x) = 2x + 4$? Justify your answer.
2. Is $g(x) = 4x + 24$ the inverse of $f(x) = \frac{1}{4}x + 6$? Justify your answer.
3. Is $h(x) = x^2 - 2$ the inverse of $g(x) = \sqrt{x + 2}$? Justify your answer.
4. Is $h(x) = x^2$ the inverse of $g(x) = \sqrt{x}$? Justify your answer.

Find the inverse of the given function.

5. $f = \{(1,3), (2,-5), (3,6)\}$
6. $g = \{(-4,1), (-3,2), (0,0), (1,10)\}$
7. $h = \{(-1,-1), (0,0), (3,3), (6,6)\}$
8.

x	*y*
−3	−2
−1	2
0	4
1	6
3	8

9.

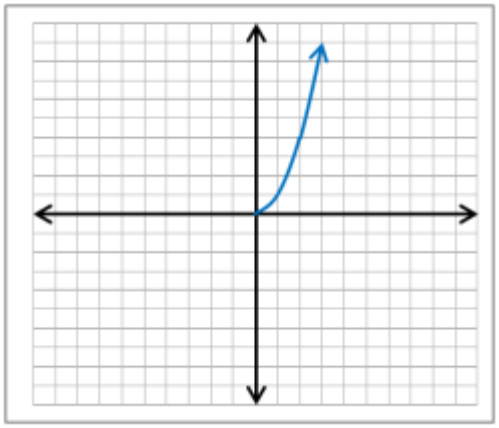
x	*y*
−3	0
1	2
6	3
13	4
22	5

10. $f(x) = 3x - 7$
11. $g(x) = -4x + 5$
12. $h(x) = \frac{2}{5}x + 6$
13. $f(x) = \frac{3x+4}{7}$
14. $g(x) = \frac{1}{4}x + 6$
15. $g(x) = -3x - 10$
16. $f(x) = \sqrt{x - 4}$
17. $g(x) = \sqrt{2x + 8}$
18. $h(x) = \sqrt{3x} - 6$
19. $f(x) = 4\sqrt{x}$
20. Graph the inverse of $f(x) = 4x - 12$.

Challenge Problems

1. Graph the inverse of $f(x) = \sqrt{x+1}$ (Hint: identify the domain of $f(x)$).

2. Graph the inverse of the function graphed below.



scale of x and y-axes is 1.

3. Find the inverse of the function $f(x) = \sqrt[3]{x+4}$.
4. Find the error in the student's work for the following problem:

   If $f(x) = \frac{x-7}{x}$, find $f^{-1}(x)$.

   Given $f(x) = \frac{x-7}{x}$

   Step 1 $y = \frac{x-7}{x}$

   Step 2 $x = \frac{y-7}{x}$

   Step 3 $x(x) = \left(\frac{y-7}{x}\right)x$

   Step 4 $x^2 = y-7$
   $+7 \quad +7$

   Step 5 $x^2 + 7 = y$

   Step 6 $f^{-1}(x) = x^2 + 7$

5. Find the inverse of the function $f(x) = \frac{x-2}{x}$.

 Name_____

Function Inverses

Date_____ Period____

State if the given functions are inverses.

1) $g(x) = 4 - \frac{3}{2}x$

$f(x) = \frac{1}{2}x + \frac{3}{2}$

2) $g(n) = \frac{-12 - 2n}{3}$

$f(n) = \frac{-5 + 6n}{5}$

3) $f(n) = \frac{-16 + n}{4}$

$g(n) = 4n + 16$

4) $f(x) = -\frac{4}{7}x - \frac{16}{7}$

$g(x) = \frac{3}{2}x - \frac{3}{2}$

5) $f(n) = -(n + 1)^3$

$g(n) = 3 + n^3$

6) $f(n) = 2(n - 2)^3$

$g(n) = \frac{4 + \sqrt[3]{4n}}{2}$

7) $f(x) = \frac{4}{-x - 2} + 2$

$h(x) = -\frac{1}{x + 3}$

8) $g(x) = -\frac{2}{x} - 1$

$f(x) = -\frac{2}{x + 1}$

Find the inverse of each function.

9) $h(x) = \sqrt[3]{x} - 3$

10) $g(x) = \frac{1}{x} - 2$

11) $h(x) = 2x^3 + 3$

12) $g(x) = -4x + 1$

13) $g(x) = \dfrac{7x + 18}{2}$

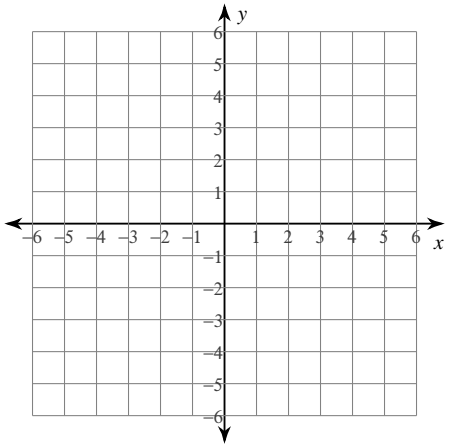
14) $f(x) = x + 3$

15) $f(x) = -x + 3$

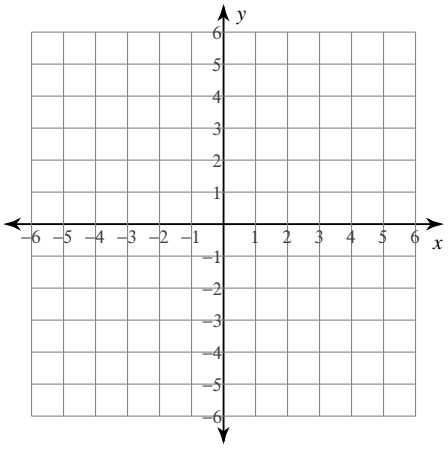
16) $f(x) = 4x$

Find the inverse of each function. Then graph the function and its inverse.

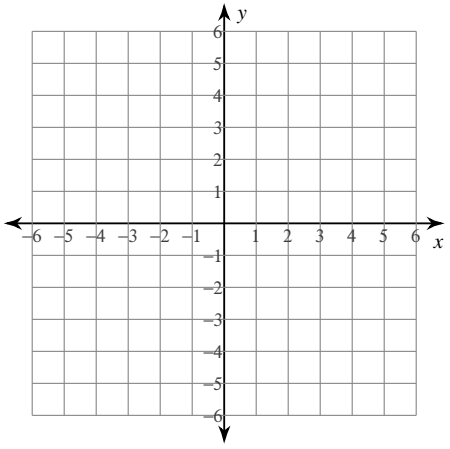
17) $f(x) = -1 - \dfrac{1}{5}x$



18) $g(x) = \dfrac{1}{x - 1}$



19) $f(x) = -2x^3 + 1$



20) $g(x) = \dfrac{-x - 5}{3}$