Name ______________________________ Class ____________ Date ____________

7-1 Reteaching

Exploring Exponential Models

- The general form of an exponential function is $y = ab^x$, where a is the initial amount and b is the growth or decay factor.
- To find b, use the formula $b = 1 + r$, where r is the constant rate of growth or decay. If r is a rate of growth, it will be positive. If r is a rate of decay, it will be negative. Therefore, if b is greater than 1, the function models growth. If b is between zero and 1, the function models decay. When you see words like *increase* or *appreciation*, think growth. When you see words like *decrease* or *depreciation*, think decay.
- For an exponential function, the y-intercept is always equal to the value of a.

Problem

Carl's weight at 12 yr is 82 lb. Assume that his weight increases at a rate of 16% each year. Write an exponential function to model the increase. What is his weight after 5 years?

Step 1 Find a and b.

$a = 82$ — a is the original amount.

$b = 1 + 0.16$ — b is the growth or decay factor. Since this problem models growth, r will be positive. Make sure to rewrite the rate, r, as a decimal.

$= 1.16$

Step 2 Write the exponential function.

$y = ab^x$ — Use the formula.

$y = 82(1.16)^x$ — Substitute.

Step 3 Calculate.

$y = 82(1.16)^5$ — Substitute 5 for x.

$y \approx 172.228$ — Use a calculator.

Carl will weigh about 172 lb in 5 years.

Exercises

Determine whether the function represents exponential growth or exponential decay. Then find the y-intercept.

1. $y = 8000(1.15)^x$

2. $y = 20(0.75)^x$

3. $y = 15\left(\frac{1}{2}\right)^x$

4. $f(x) = 6\left(\frac{5}{2}\right)^x$

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7-1 Reteaching (continued)

Exploring Exponential Models

You can use the general form of an exponential function to solve word problems involving growth or decay.

Problem

A motorcycle purchased for \$9000 today will be worth 6% less each year. How much will the motorcycle be worth at the end of 5 years?

Step 1 Find a and b.

$a = 9000$	a is the original amount.
$b = 1 + (-0.06)$	b is the growth or decay factor. Since this problem models decay, r will be negative. Make sure to rewrite the rate, r, as a decimal.
$= 0.94$	

Step 2 Write the exponential function.

$y = ab^x$	Use the formula.
$y = 9000(0.94)^x$	Substitute.

Step 3 Calculate.

$y = 9000(0.94)^5$	Substitute 5 for x.
$y \approx 6605.13$	Use a calculator.

The motorcycle will be worth about \$6605.13 after 5 years.

Exercises

Write an exponential function to model each situation. Find each amount after the specified time.

5. A tree 3 ft tall grows 8% each year. How tall will the tree be at the end of 14 yr? Round the answer to the nearest hundredth.

6. The price of a new home is \$126,000. The value of the home appreciates 2% each year. How much will the home be worth in 10 yr?

7. A butterfly population is decreasing at a rate of 0.82% per year. There are currently about 100,000 butterflies in the population. How many butterflies will there be in the population in 250 years?

8. A car depreciates 10% each year. If you bought this car today for \$5000, how much will it be worth in 7 years?

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Name ______________________ Class ____________ Date ____________

7-2 Reteaching

Properties of Exponential Functions

There are four types of transformations that can change the graph of an exponential function.

Stretches

The factor a in $y = ab^x$ can stretch the graph of an exponential function when

$$|a| > 1$$

Compressions

The factor a in $y = ab^x$ can compress the graph of an exponential function when

$$0 < |a| < 1$$

Reflections

The factor a in $y = ab^x$ can reflect the graph of an exponential function in the x-axis when

$$a < 0$$

Translations

The graph of an exponential function translates horizontally by h; vertically by k.

$$y = ab^{(x-h)} + k$$

Problem

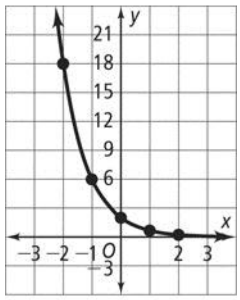
How does the graph of $y = 2\left(\frac{1}{3}\right)^{x+1} - 4$ compare to the parent function $y = 2\left(\frac{1}{3}\right)^x$?

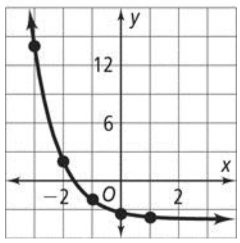
Step 1 Determine the base of the function $y = 2\left(\frac{1}{3}\right)^x$. Because $b < 1$, the graph will represent exponential decay.

Step 2 Make a table. Find more values if necessary to get a good picture of the graph.

x	$y = 2\left(\frac{1}{3}\right)^x$	y
–2	$2\left(\frac{1}{3}\right)^{-2} = 2(9)$	18
–1	$2\left(\frac{1}{3}\right)^{-1} = 2(3)$	6
0	$2\left(\frac{1}{3}\right)^{0} = 2(1)$	2
1	$2\left(\frac{1}{3}\right)^{1} = 2\left(\frac{1}{3}\right)$	$\frac{2}{3}$
2	$2\left(\frac{1}{3}\right)^{2} = 2\left(\frac{1}{9}\right)$	$\frac{2}{9}$

Step 3 Use the values for x and y from the table to graph the function.





Step 4 For $y = 2\left(\frac{1}{3}\right)^{x+1} - 4$, $h = -1$ and $k = -4$. Shift the graph of the parent function above 1 unit left and 4 units down. The horizontal asymptote shifts down as well, from $y = 0$ to $y = -4$.

Step 5 Use a graphing calculator to check your graph.

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Name ______________________ Class __________ Date __________

7-2 Reteaching (continued)

Properties of Exponential Functions

For problems involving continuously compounded interest, use the following formula:

Continuously Compounded Interest $A(t) = p.e^{rt}$

$A(t)$ is the amount in account after time t.

P is the principal.

r is the annual interest rate (as a decimal).

t is time (in years).

Problem

Suppose you invest \$2000 at an annual interest rate of 5.5% compounded continuously. How much will you have in the account in 10 years?

What do you know?

principle P = \$2000
interest rate r = 5.5% = 0.055
time t = 10 years

Use the formula.

$$A(t) = P \cdot e^{rt}$$
$$= 2000 \cdot e^{(0.055)(10)}$$
$$= 2000 \cdot e^{0.55}$$
$$\approx 3466.50$$

In ten years, you will have \$3466.50.

Exercises

Graph each exponential function.

1. $y = \left(\frac{1}{5}\right)^x$

2. $y = 3 + 1$

3. $y = 5^x$

4. $y = -\left(\frac{1}{2}\right)^x$

5. $y = -\left(\frac{1}{2}\right)^x + 4$

6. $y = \left(\frac{1}{4}\right)^x$

7. $y = \left(\frac{1}{4}\right)^{x-1}$

8. $y = 4^x + 1$

9. $y = -(2)^x$

10. Suppose you invest \$7500 at an annual interest of 7% compounded continuously.

a. How much will you have in the account in 10 years?

b. How long will it take for the account to reach \$20,000?

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Name ______________________________ Class ____________ Date ____________

7-3 Reteaching

Logarithmic Functions as Inverses

A logarithmic function is the inverse of an exponential function.

To evaluate logarithmic expressions, use the fact that $x = \log_b y$ is the same as $y = b^x$. Keep in mind that $x = \log y$ is another way of writing $x = \log_{10} y$.

Problem

What is the logarithmic form of $6^3 = 216$?

Step 1 Determine which equation to use.

The equation is in the form $b^x = y$.

Step 2 Find x, y, and b.

$b = 6$, $x = 3$, and $y = 216$

Step 3 Because $y = b^x$ is the same as $x = \log_b y$, rewrite the equation in logarithmic form by substituting for x, y, and b.

$3 = \log_6 216$

Exercises

Write each equation in logarithmic form.

1. $4^{-3} = \frac{1}{64}$

2. $5^{-2} = \frac{1}{25}$

3. $8^{-1} = \frac{1}{8}$

4. $11^0 = 1$

5. $6^1 = 6$

6. $6^{-3} = \frac{1}{216}$

7. $17^0 = 1$

8. $17^1 = 17$

Problem

What is the exponential form of $4 = \log_5 625$?

Step 1 Determine which equation to use.

The equation is in the form $x = \log_b y$.

Step 2 Find x, y, and b.

$x = 4$, $b = 5$, and $y = 625$

Step 3 Because $x = \log_b y$ is the same as $y = b^x$, rewrite the equation in exponential form by substituting for x, y, and b.

$625 = 5^4$

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7-3 Reteaching (continued)

Logarithmic Functions as Inverses

Exercises

Write each equation in exponential form.

9. $3 = \log_2 8$

10. $2 = \log_5 25$

11. $\log 0.1 = -1$

12. $\log 7 \approx 0.845$

13. $\log 1000 = 3$

14. $-2 = \log 0.01$

15. $\log_3 81 = 4$

16. $\log_{49} 7 = \frac{1}{2}$

17. $\log_8 \frac{1}{4} = -\frac{2}{3}$

18. $\log_2 128 = 7$

19. $\log_5 \frac{1}{625} = -4$

20. $\log_6 36 = 2$

Problem

What is the value of $\log_4 32$?

$x = \log_4 32$	Write the equation in logarithmic form $x = \log_b y$.
$32 = 4^x$	Rewrite in exponential form $y = b^x$.
$2^5 = (2^2)^x$	Rewrite each side of the equation with like bases in order to solve the equation.
$2^5 = 2^{2x}$	Simplify.
$5 = 2x$	Set the exponents equal to each other.
$x = \frac{5}{2}$	Solve for *x*.
$\log_4 32 = \frac{5}{2}$	

Exercises

Evaluate the logarithm.

21. $\log_2 64$

22. $\log_2 64$

23. $\log_3 3^4$

24. $\log 10$

25. $\log 0.1$

26. $\log 1$

27. $\log_8 2$

28. $\log_{32} 2$

29. $\log_9 3$

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Name ______________________ Class ____________ Date ____________

7-4 Reteaching

Properties of Logarithms

You can write a logarithmic expression containing more than one logarithm as a single logarithm as long as the bases are equal. You can write a logarithm that contains a number raised to a power as a logarithm with the power as a coefficient. To understand the following properties, remember that logarithms are powers.

Name	Formula	Why?
Product Property	$\log_b mn = \log_b m + \log_b n$	When you multiply two powers, you add the exponents. Example: $2^6 \cdot 2^2 = 2^{(6+2)} = 2^8$
Quotient Property	$\log_b \frac{m}{n} = \log_b m + \log_b n$	When you divide two powers, you subtract the exponents. Example: $\frac{2^6}{2^2} = 2^{(6-2)} = 2^4$
Power Property	$\log_b m^n = n\log_b m$	When you raise a power to a power, you multiply the exponents. Example: $(2^6)^2 = 2^{(6\cdot 2)} = 2^{12}$

Problem

What is $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27$ written as a single logarithm?

$2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 6^2 - \log_2 9 + \log_2 27^{\frac{1}{3}}$ Use the Power Property twice.

$= \log_2 36 - \log_2 9 + \log_2 3$ $6^2 = 36, 27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

$= (\log_2 36 - \log_2 9) + \log_2 3$ Group two of the logarithms. Use order of operations.

$= \log_2 \frac{32}{9} + \log_2 3$ Quotient Property

$= \log_2 \left(\frac{36}{9} \bullet 3\right)$ Product Property

$= \log_2 12$ Simplify.

As a single logarithm, $2\log_2 6 - \log_2 9 + \frac{1}{3}\log_2 27 = \log_2 12$.

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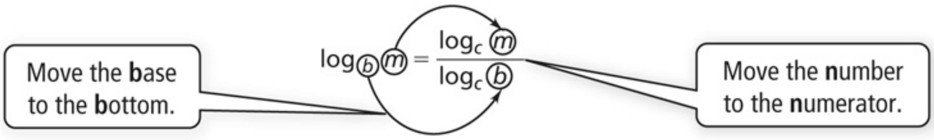
Name ______________________ Class ____________ Date ____________

7-4 Reteaching (continued)

Properties of Logarithms

To evaluate logarithms with any base, you can rewrite the logarithm as a quotient of two logarithms with the same base.



Problem

What is $\log_4 8$ written as a quotient of two logarithms with base 2? Simplify your answer, if possible.

$\log_4 8 = \frac{\log_2 8}{\log_2 4}$ The base is 4 and the number is 8. Move the base to the bottom and the number to the numerator.

$= \frac{3}{2}$ Evaluate the logarithms in the numerator and the denominator.

Exercises

Write each logarithmic expression as a single logarithm.

1. $\log_3 13 + \log_3 3$ **2.** $2 \log x + \log 5$ **3.** $\log_4 2 - \log_4 6$

4. $3\log_3 3 - \log_3 3$ **5.** $\log_5 8 + \log_5 x$ **6.** $\log 2 - 2 \log x$

7. $\log_2 x + \log_2 y$ **8** $3\log_7 x - 5\log_7 y$ **9.** $4 \log x + 3 \log x$

10. $\log_5 x + 3\log_5 y$ **11.** $3\log_2 x - \log_2 y$ **12.** $\log_2 16 - \log_2 8$

Write each logarithm as a quotient of two common logarithms. Simplify your answer, if possible. (*Hint:* Common logarithms are logarithms with base 10.)

13. $\log_4 12$ **14.** $\log_2 1000$ **15.** $\log_5 16$

16. $\log_{11} 205$ **17.** $\log_9 32$ **18.** $\log_{100} 5$

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Name______________________ Class __________ Date__________

7-5 Reteaching

Exponential and Logarithmic Equations

Use logarithms to solve exponential equations.

Problem

What is the solution of $7 - 5^{2x-1} = 4$?

$7 - 5^{2x-1} = 4$	
$-5^{2x-1} = -3$	First isolate the term that has the variable in the exponent. Begin by subtracting 7 from each side.
$5^{2x-1} = 3$	Multiply each side by –1.
$\log_5 5^{2x-1} = \log_5 3$	Because the variable is in the exponent, use logarithms. Take $\log_5$ of each side because 5 is the base of the exponent.
$(2x - 1)\log_5 5 = \log_5 3$	Use the Power Property of Logarithms.
$2x - 1 = \log_5 3$	Simplify. (Recall that $\log_b b = 1$.)
$2x - 1 = \dfrac{\log 3}{\log 5}$	Apply the Change of Base Formula.
$2x = \dfrac{\log 3}{\log 5} + 1$	Add 1 to each side.
$x = \dfrac{1}{2}\left(\dfrac{\log 3}{\log 5} + 1\right)$	Divide each side by 2.
$x \approx 0.84$	Use a calculator to find a decimal approximation.

Exercises

Solve each equation. Round the answer to the nearest hundredth.

1. $2^x = 5$

2. $10^{2x} = 8$

3. $5^{x+1} = 25$

4. $2^{x+3} = 9$

5. $3^{2x-3} = 7$

6. $4^x - 5 = 3$

7. $5 + 2^{x+6} = 9$

8. $4^{3x} + 2 = 3$

9. $1 - 3^{2x} = -5$

10. $2^{3x} - 2 = 13$

11. $5^{2x+7} - 1 = 8$

12. $7 - 2^{x+7} = 5$

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7-5 Reteaching (continued)

Exponential and Logarithmic Equations

Use exponents to solve logarithmic equations.

Problem

What is the solution of $8 - \log(4x - 3) = 4$?

$8 - 2\log(4x - 3) = 4$

$-\log(4x - 3) = -4$	First isolate the term that has the variable in the logarithm. Begin by subtracting 8 from each side.
$\log(4x - 3) = 4$	Multiply each side by –1.
$4x - 3 = 10^4$	Write in exponential form.
$4x - 3 = 10{,}000$	Simplify.
$4x = 10{,}003$	Add 3 to each side.
$x = \frac{10{,}003}{4}$	Solve for *x*.
$x = 2500.75$	Divide.

Exercises

Solve each equation. Round the answer to the nearest thousandth.

13. $\log x = 2$

14. $\log 3x = 3$

15. $\log 2x + 2 = 6$

16. $5 + \log(2x + 1) = 6$

17. $\log 5x + 62 = 62$

18. $6 - \log\frac{1}{2}x = 3$

19. $\log(4x - 3) + 6 = 4$

20. $\frac{2}{3}\log 5x = 2$

21. $2\log 250x - 6 = 4$

22. $5 - 2\log x = \frac{1}{2}$

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Name ______________________ Class ____________ Date ____________

7-6 Reteaching

Natural Logarithms

The **natural logarithmic function** is a logarithm with base e, an irrational number.

You can write the natural logarithmic function as $y = \log_e x$, but you usually write it as $y = \ln x$.

$y = e^x$ and $y = \ln x$ are inverses, so if $y = e^x$, then $x = \ln y$.

To solve a natural logarithm equation:

- If the term containing the variable is an exponential expression, rewrite the equation in logarithmic form.
- If term containing the variable is a logarithmic expression, rewrite the equation in exponential form.

Problem

What is the solution of $4e^{2x} - 2 = 3$?

Step 1 Isolate the term containing the variable on one side of the equation.

$$4e^{2x} - 2 = 3$$

$4e^{2x} = 5$ Add 2 to each side of the equation.

$e^{2x} = \frac{5}{4}$ Divide each side of the equation by 4.

Step 2 Take the natural logarithm of each side of the equation.

$$\ln(e^{2x}) = \ln\left(\frac{5}{4}\right)$$

$2x = \ln\left(\frac{5}{4}\right)$ Definition of natural logarithm

Step 3 Solve for the variable.

$x = \frac{\ln\left(\frac{5}{4}\right)}{2}$ Divide each side of the equation by 2.

$x \approx 0.112$ Use a calculator.

Step 4 Check the solution.

$$4e^{2(0.112)} - 2 \stackrel{?}{=} 3$$

$$4e^{0.224} - 2 \stackrel{?}{=} 3$$

$$3.004 \approx 3$$

The solution is $x \approx 0.112$.

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7-6 Reteaching (continued)

Natural Logarithms

Problem

What is the solution of $\ln(t-2)^2 + 1 = 6$? Round your answer to the nearest thousandth.

Step 1 Isolate the term containing the variable on one side of the equation.

$\ln(t-2)^2 + 1 = 6$

$\ln(t-2)^2 = 5$ Subtract 1 from each side of the equation.

Step 2 Raise each side of the equation to the base *e*.

$e^{\ln}(t-2)^2 = e^5$

$(t-2)^2 = e^5$ Definition of natural logarithm

Step 3 Solve for the variable.

$t-2 = \pm e^{\frac{5}{2}}$ Take the square root of each side of the equation.

$t = 2 \pm e^{\frac{5}{2}}$ Add 2 to each side of the equation.

$t \approx 14.182$ or -10.182 Use a calculator.

Step 4 Check the solution.

$\ln(14.182-2)^2 \stackrel{?}{=} 5$	$\ln(-10.182-2)^2 \stackrel{?}{=} 5$
$4.9999 \approx 5$	$4.9999 \approx 5$

The solutions are $t \approx 14.182$ and -10.182.

Exercises

Use natural logarithms to solve each equation. Round your answer to the nearest thousandth. Check your answers.

1. $2e^x = 4$ **2.** $e^{4x} = 25$ **3.** $e^x = 72$

4. $e^{3x} = 124$ **5.** $12e^{3x-2} = 8$ **6.** $\frac{1}{2}e^{6x} = 5$

Solve each equation. Round your answer to the nearest thousandth. Check your answers.

7. $\ln(x-3) = 2$ **8.** $\ln 2t = 4$ **9.** $1 + \ln x^2 = 2$

10. $\ln(2x-5) = 3$ **11.** $\frac{1}{3}\ln 2t = 1$ **12.** $\ln(t-4)^2 + 2 = 5$

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