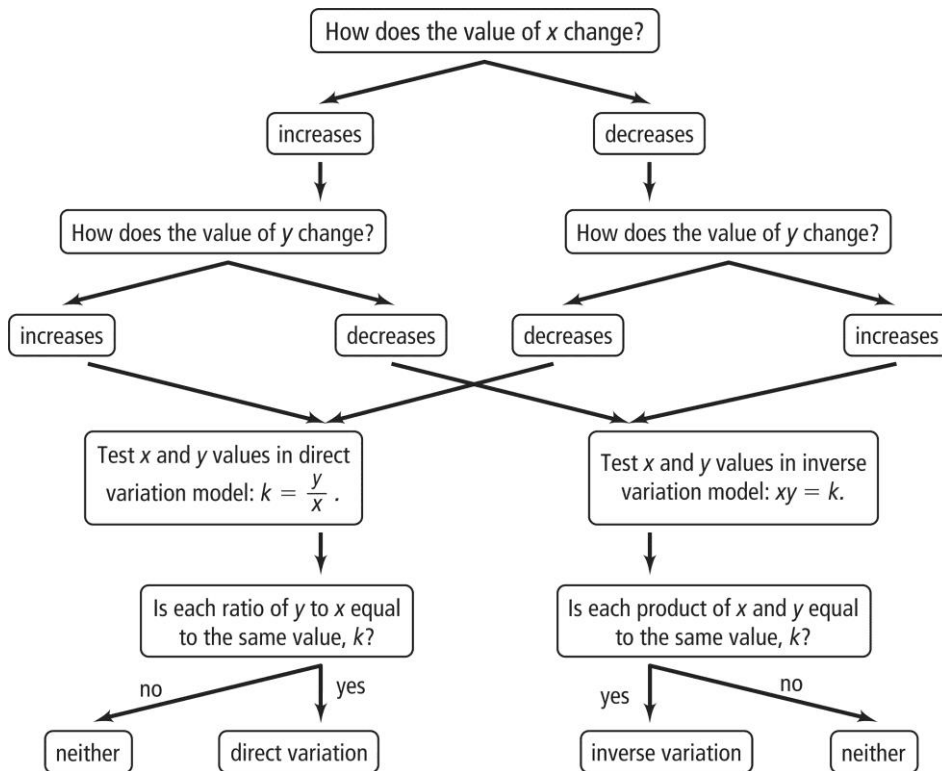


8-1

Reteaching

Inverse Variation

The flowchart below shows how to decide whether a relationship between two variables is a direct variation, inverse variation, or neither.



Problem

Do the data in the table represent a direct variation, inverse variation, or neither?

x	1	2	4	5
y	20	10	5	4

As the value of x increases, the value of y decreases, so test the table values in the inverse variation model: $xy = k$: $1 \cdot 20 = 20$, $2 \cdot 10 = 20$, $4 \cdot 5 = 20$, $5 \cdot 4 = 20$. Each product equals the same value, 20, so the data in the table model an inverse variation.

Exercises

Do the data in the table represent a direct variation, inverse variation, or neither?

1.

x	5	10	15	20
y	10	20	30	40

2.

x	1	3	4	6
y	12	4	3	2

8-1**Reteaching** (continued)

Inverse Variation

To solve problems involving inverse variation, you need to solve for the constant of variation k before you can find an answer.

Problem

The time t that is necessary to complete a task varies inversely as the number of people p working. If it takes 4 h for 12 people to paint the exterior of a house, how long does it take for 3 people to do the same job?

$$t = \frac{k}{p}$$

Write an inverse variation. Because time is dependent on people, t is the dependent variable and p is the independent variable.

$$4 = \frac{k}{12}$$

Substitute 4 for t and 12 for p .

$$48 = k$$

Multiply both sides by 12 to solve for k , the constant of variation.

$$4 = \frac{48}{p}$$

Substitute 48 for k . This is the equation of the inverse variation.

$$t = \frac{48}{3} = 16$$

Substitute 3 for p . Simplify to solve the equation.

It takes 3 people 16 h to paint the exterior of the house.

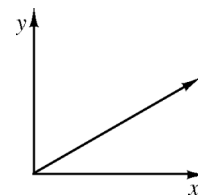
Exercises

- The time t needed to complete a task varies inversely as the number of people p . It takes 5 h for seven men to install a new roof. How long does it take ten men to complete the job?
- The time t needed to drive a certain distance varies inversely as the speed r . It takes 7.5 h at 40 mi/h to drive a certain distance. How long does it take to drive the same distance at 60 mi/h?
- The cost of each item bought is inversely proportional to the number of items when spending a fixed amount. When 42 items are bought, each costs \$1.46. Find the number of items when each costs \$2.16.
- The length l of a rectangle of a certain area varies inversely as the width w . The length of a rectangle is 9 cm when the width is 6 cm. Determine the length if the width is 8 cm.

DIRECT, JOINT, AND INVERSE VARIATION

DIRECT VARIATION

A linear equation of the form $y = kx$ with $k \neq 0$ is called **direct variation**. The variable y varies directly with the variable x . In other words, the variables change at the same rate: If the value of one variable doubles, the value of other variable doubles, if one value triples, the other triples. The graph is a line passing through the origin with slope k . In this case the slope is also called the **constant of variation**.



Direct Variation

Example 1

The amount of money spent at the gas station varies directly with the number of gallons purchased. When 11.5 gallons of gas was purchased the cost was \$37.72. Determine k , the constant of variation and the equation that represents this situation.

If $y =$ the total cost and $x =$ number of gallons, the general equation is $y = kx$. Substitute the given values and solve.

$$\begin{aligned}y &= kx \\37.72 &= k(11.5) \\k &= 3.28\end{aligned}$$

The equation is $y = 3.28x$.

Example 2

The number of bricks laid varies directly with the amount of time spent. If 45 bricks are laid in 65 minutes, determine the equation that represents this situation. Also determine the time it would take to lay 500 bricks.

If $b =$ number of bricks and $t =$ time, the general equation is $b = kt$. Substitute the given values and solve.

$$\begin{aligned}b &= kt \\45 &= k(65) \\k &= \frac{45}{65} = \frac{9}{13}\end{aligned}$$

The equation is $b = \frac{9}{13}t$.

To find the time to lay 500 bricks, substitute 500 for b and solve.

$$\begin{aligned}500 &= \frac{9}{13}t \\t &= \frac{13}{9}(500) \approx 722 \text{ min.} \approx 12 \text{ hr.}\end{aligned}$$

Problems

Write an equation of variation to represent the situation and solve for the indicated information.

1. The number of centimeters y in a linear measurement varies directly with the number of inches x in the measurement. Pablo's height is 152.4 centimeters or 60 inches. What is Maria's height in centimeters if she is 64 inches tall?
2. The number of gallons g of fuel used on a trip varies directly with the number of miles m traveled. If a trip of 270 miles required 12 gallons of fuel, how many gallons are required for a trip of 400 miles?
3. Karen earns \$28.50 for working six hours. If the amount m she earns varies directly with h the number of hours she works, how much will she earn for working 10 hours?
4. A bottle of 150 vitamins costs \$5.25. If the cost varies directly with the number of vitamins in the bottle, what should a bottle with 250 vitamins cost?
5. Wei received \$55.35 in interest on the \$1230 in her credit union account. If the interest varies directly with the amount deposited, how much would Wei receive for the same amount of time if she had \$2000 in the account?

JOINT VARIATION

There are situations when more than one variable is involved in a direct variation problem. In these cases the problem is called **joint variation**. The equation remains the same except that additional variables are included in the product. For example if A varies jointly with the values of b and h the equation will be $A = kbh$.

Example

Variable I varies jointly as the values of P and T . If $I = 1200$ when $P = 5000$ and $T = 3$, find I when $P = 7500$ and $T = 4$.

Use the same method as above.

Write the general equation.

$$I = kPT$$

Solve for k (the constant of variation)

$$1200 = k(5000)(3)$$

$$k = 0.08$$

Use the formula, k , and the new values.

$$I = 0.08(7500)(4) = 2400$$

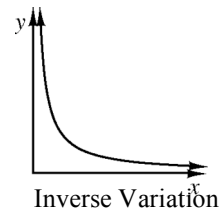
Problems

Write an equation of variation to represent the situation and solve for the missing information.

- The cost c of materials for a deck varies jointly with the width w and the length l . If $c = \$470.40$ when $w = 12$ and $l = 16$, find the cost when $w = 10$ and $l = 25$.
- The value of real estate V varies jointly with the neighborhood index N and the square footage of the house S . If $V = \$376,320$ when $N = 96$ and $S = 1600$, find the value of a property with $N = 83$ and $S = 2150$.
- The number of gallons g in a circular swimming pool varies jointly with the square of the radius r^2 and the depth d . If $g = 754$ when $r = 4$ and $d = 2$, find the number of gallons in the pool when $r = 3$ and $d = 1.5$.

INVERSE VARIATION

An equation of the form $y = \frac{k}{x}$ or $xy = k$ ($k \neq 0$) is called **inverse variation**. In this situation, as the variable x increases, the variable y decreases and the reverse is also true. The graph is a curve that never touches either axis.



Example

The time it takes to travel a fixed distance varies inversely with the speed traveled. If it takes Pam 40 minutes to bike to the secret fishing spot at 9 miles per hour, what is the equation that represents this situation? How long will it take if she rides 12 miles per hour?

Since the units are not the same first change 40 minutes into hours: $\frac{40}{60} = \frac{2}{3}$ hour.

Letting $y =$ time and $x =$ speed, use the equation: $y = \frac{k}{x}$ (time varies inversely with speed)

Substitute the values and solve for k . $\frac{2}{3} = \frac{k}{9}$, so $k = 6$.

The equation of this inverse variation is: $y = \frac{6}{x}$

If she rides 12 miles per hour the time is: $y = \frac{6}{12} = \frac{1}{2}$ hour or 30 minutes

Problems

Write an equation of variation to represent the situation and solve for the missing information.

- The volume V of a gas kept at a constant temperature varies inversely as the pressure p . If the pressure is 24 pounds per square inch, the volume is 15 cubic feet. What will be the volume when the pressure is 30 pounds per square inch?
- The time to complete a project varies inversely with the number of employees. If 3 people can complete the project in 7 days, how long will it take 5 people?
- The time needed to travel a certain distance varies inversely with the rate of speed. If it takes 8 hours to travel a certain distance at 36 miles per hour, how long will it take to travel the same distance at 60 miles per hour?
- The number of revolutions made by a tire traveling over a fixed distance varies inversely to the radius of the tire. A 12-inch radius tire makes 100 revolutions to travel a certain distance. How many revolutions would a 16-inch radius tire require to travel the same distance?

13. For a fixed number of miles, the gas mileage of a car (miles/gallon) varies inversely with the number of gallons used. One year an employee driving a truck averaged 24 miles per gallon and used 750 gallons of gas. If the next year, to drive the same number of miles the employee drove a compact car averaging 39 miles per gallon, how many gallons of gas would be used?

(The following are extensions and a mixture of the previous types.)

14. To build a sound wall along the highway, the amount of time t needed varies **directly** with the number of cement blocks c needed and **inversely** with the number of workers w . A sound wall made of 2400 blocks, using six workers takes 18 hours to complete. How long would it take to build a wall of 4500 blocks with 10 workers? HINT: $t = \frac{kc}{w}$
15. The time needed to paint a fence varies directly with the length of the fence and indirectly with the number of painters. If it takes five hours to paint 200 feet of fence with three painters, how long will it take five painters to paint 500 feet of fence?
16. The time to prepare a field for planting is inversely proportional to number of people who are working. A large field can be prepared by five workers in 24 days. In order to finish the field sooner, the farmer plans to hire additional workers. How many workers are needed to finish the field in 15 days?
17. An egg is dropped from the roof of a building. The distance it falls varies directly with the square of the time it falls. If it takes $\frac{1}{2}$ second for the egg to fall eight feet, how long will it take the egg to fall 200 feet?
18. The number of hours needed to assemble computers varies directly as the number of computers and inversely as the number of workers. If 4 workers can assemble 12 computers in 9 hours, how many workers are needed to assemble 48 computers in 8 hours?
19. The weigh of a person varies inversely as the square of the distance from the center of the earth. If the radius of the earth is 4000 miles, how much would a 180 pound person weigh, 2000 miles above the surface of the earth?
20. The strength of a rectangular beam varies jointly as its width and the square of it depth. If the strength of a beam three inches wide by 10 inches deep is 1200 pounds per square inch, what is the strength of a beam four inches wide and six inches deep?

Rational Expressions

State the excluded values for each.

1) $\frac{60x^3}{12x}$

2) $\frac{70v^2}{100v}$

3) $\frac{m+7}{m^2+4m-21}$

4) $\frac{n^2+6n+5}{n+1}$

5) $\frac{35x-35}{25x-40}$

6) $\frac{-n^2+16n-63}{n^2-2n-35}$

Simplify each and state the excluded values.

7) $\frac{p+4}{p^2+6p+8}$

8) $\frac{9}{15a-15}$

9) $\frac{2a^2+10a}{3a^2+15a}$

10) $\frac{p^2-3p-10}{p^2+p-2}$

11) $\frac{x^2+x-6}{x^2+8x+15}$

12) $\frac{a^2+5a+4}{a^2+9a+20}$

13) $\frac{x^2 - 2x - 15}{x^2 - 6x + 5}$

14) $\frac{10x - 6}{10x - 6}$

15) $\frac{(v - 7)(v + 8)}{(v + 8)(v - 10)} \div \frac{1}{v - 10}$

16) $\frac{n + 3}{n + 2} \div \frac{(n - 1)(n + 3)}{(n - 1)^2}$

17) $\frac{x + 3}{4} \cdot \frac{3(x - 6)}{3(x + 3)}$

18) $\frac{x - 8}{(x + 6)(x - 8)} \cdot \frac{4x(x + 10)}{x + 10}$

19) $\frac{2b^2 - 12b}{b + 5} \div \frac{b - 6}{b + 5}$

20) $\frac{1}{n + 9} \div \frac{6 - n}{3n - 18}$

21) $\frac{28 - 7b}{b - 4} \cdot \frac{1}{b + 10}$

22) $\frac{2}{v^2 - 12v + 27} \cdot \frac{v^2 - 12v + 27}{3}$

23) $\frac{1}{5p^2} \div \frac{9p - 36}{5p^3 - 35p^2}$

24) $\frac{8 - 7x - x^2}{x + 8} \cdot \frac{x + 5}{9x - 9}$

25) $\frac{x^2 - 16}{9 - x} \cdot \frac{x^2 + x - 90}{x^2 + 14x + 40}$

26) $\frac{10x^2 - 20x}{40x^3 - 80x^2} \cdot \frac{16x^3 + 80x^2}{6x + 30}$

Adding/Subtracting Rational Expressions

Simplify each expression.

1) $\frac{u-v}{8v} + \frac{6u-3v}{8v}$

2) $\frac{m-3n}{6m^3n} - \frac{m+3n}{6m^3n}$

3) $\frac{5}{a^2+3a+2} + \frac{5a+1}{a^2+3a+2}$

4) $\frac{5}{10n^2+16n+6} + \frac{n-6}{10n^2+16n+6}$

5) $\frac{r+6}{3r-6} + \frac{r+1}{3r-6}$

6) $\frac{x+2}{2x^2+13x+20} - \frac{x+3}{2x^2+13x+20}$

7) $\frac{6}{x-1} - \frac{5x}{4}$

8) $6 - \frac{x+5}{(7x-5)(x+4)}$

9) $\frac{3}{x+7} + \frac{4}{x-8}$

10) $\frac{3}{4v^2+4v} - \frac{7}{2}$

11) $\frac{7}{3} - \frac{8}{12x-8}$

12) $\frac{5}{n+5} + \frac{4n}{2n+6}$

13) $\frac{2x}{5x+4} + \frac{6x}{2x+3}$

14) $\frac{2}{3x^2+12x} + \frac{8}{2x}$

15) $\frac{7n}{n+1} + \frac{8}{n-7}$

16) $\frac{2}{n+8} + \frac{4}{n+1}$

17) $\frac{3}{8} - \frac{3}{3x+4}$

18) $\frac{3}{b-8} + \frac{7}{b+3}$

19) $\frac{3}{x+6} + \frac{7}{x-2}$

20) $\frac{4}{x+1} - \frac{2}{x+2}$

21) $\frac{5n+5}{5n^2+35n-40} + \frac{7n}{3n}$

22) $\frac{3}{n-5} + \frac{6}{3n-8}$

23) $\frac{\frac{25}{4}}{\frac{1}{5} - \frac{4}{25}}$

24) $\frac{8}{\frac{4}{9} + \frac{16}{9}}$

25) $\frac{\frac{a}{25} - \frac{a}{5}}{a}$

26) $\frac{\frac{5}{4}}{\frac{5}{m} - \frac{4}{m}}$

Critical thinking questions:

27) Simplify: $\frac{a}{b} + \frac{c}{d}$

28) Split into a sum of two rational expressions with unlike denominators:

$$\frac{2x+3}{x^2+3x+2}$$

Graphing Rational Functions

Identify the points of discontinuity, holes, vertical asymptotes, x-intercepts, and horizontal asymptote of each.

1) $f(x) = \frac{1}{3x^2 + 3x - 18}$

2) $f(x) = \frac{x - 2}{x - 4}$

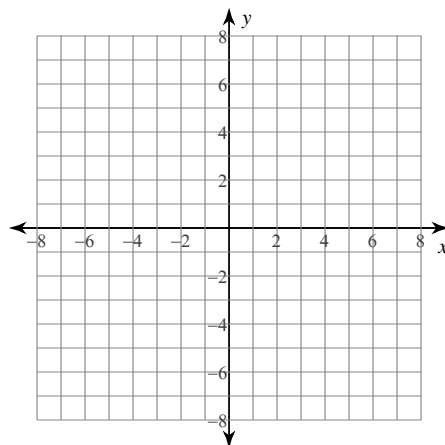
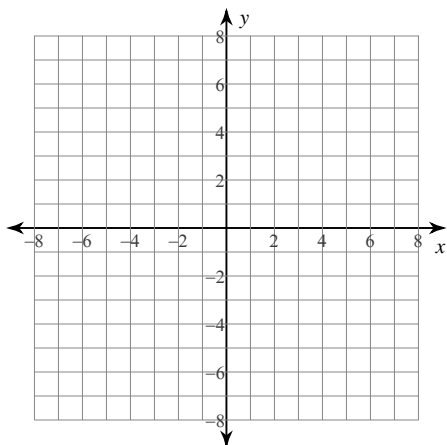
3) $f(x) = \frac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$

4) $f(x) = \frac{x^2 + x - 6}{-4x^2 - 16x - 12}$

Identify the points of discontinuity, holes, vertical asymptotes, and horizontal asymptote of each. Then sketch the graph.

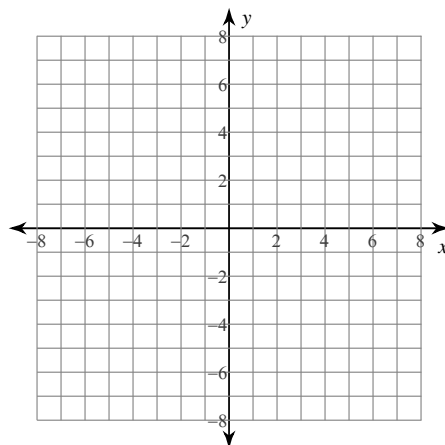
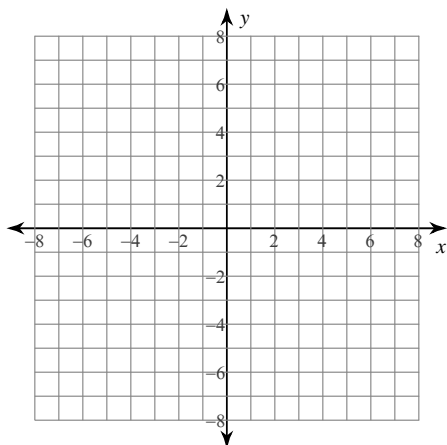
5) $f(x) = -\frac{4}{x^2 - 3x}$

6) $f(x) = \frac{x - 4}{-4x - 16}$

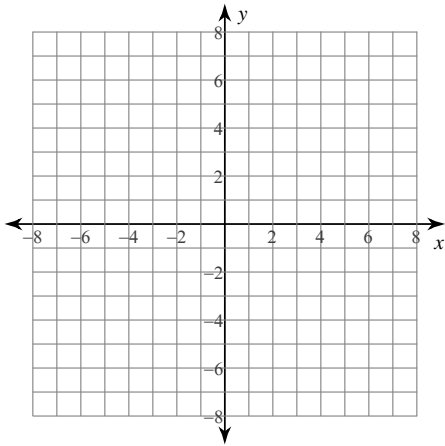


7) $f(x) = \frac{x + 4}{-2x - 6}$

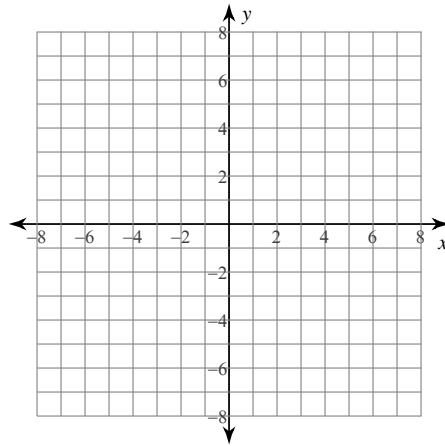
8) $f(x) = \frac{x^3 - 9x}{3x^2 - 6x - 9}$



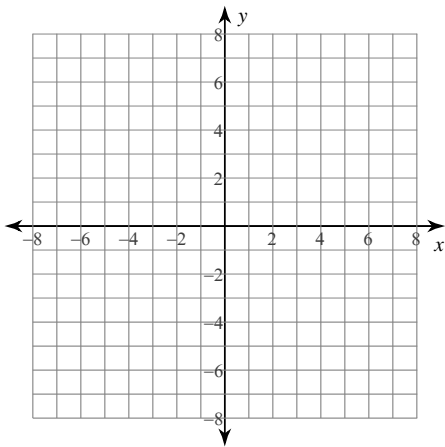
$$9) f(x) = \frac{3x^2 - 12x}{x^2 - 2x - 3}$$



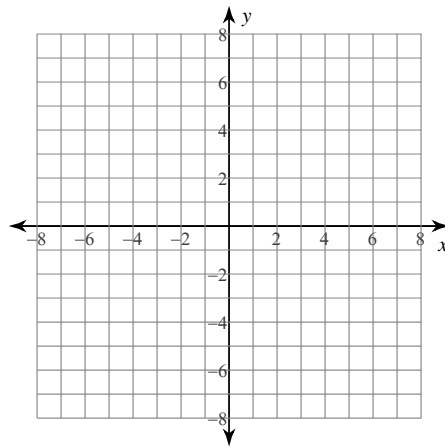
$$10) f(x) = \frac{x^3 - 16x}{-4x^2 + 4x + 24}$$



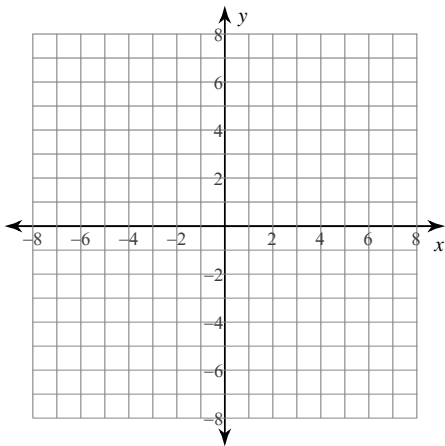
$$11) f(x) = \frac{x^2 + 2x}{-4x + 8}$$



$$12) f(x) = \frac{x + 2}{2x + 6}$$



$$13) f(x) = \frac{2x^2 + 10x + 12}{x^2 + 3x + 2}$$



$$14) f(x) = \frac{3}{x - 2}$$

